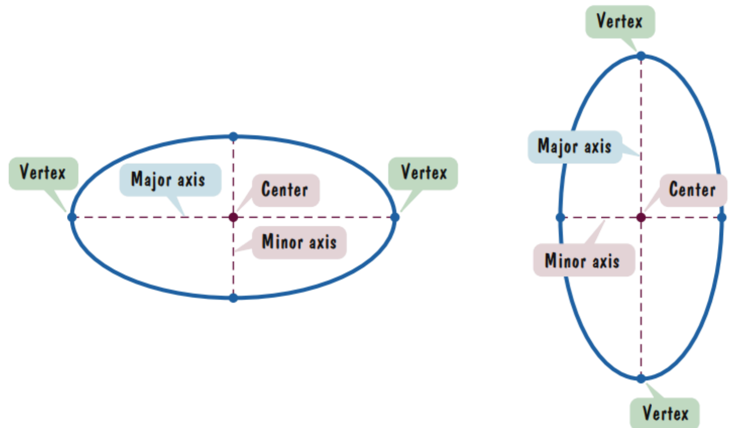
Period 4, April 4, 2025



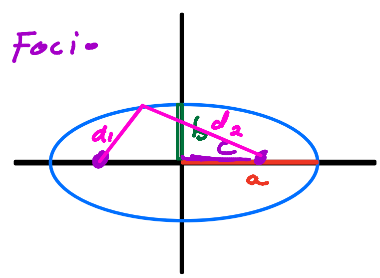
A BRIEF REVIEW The Distance Formula

The distance, d, between the points (x_1, y_1) and (x_2, y_2) in the rectangular coordinate system is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

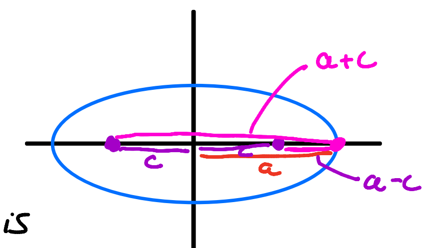
For more detail, see Section 1.9, Objective 1.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

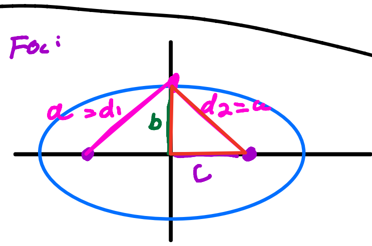


$d_1 + d_2 = \text{constant}$

$a + c + a - c = 2a = \text{constant}$



Foci are always on major axis
(bigger axis)



$d_1 = d_2$ $\quad d_1 + d_2 = 2a$

$a = a$

$c^2 + b^2 = a^2$

$\text{Semi major}^2 = (\text{Foci Length})^2 + \text{semi minor}^2$

Standard Forms of the Equations of an Ellipse

The **standard form of the equation of an ellipse** with center at the origin, and major and minor axes of lengths $2a$ and $2b$ (where a and b are positive, and $a^2 > b^2$) is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{or} \qquad \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1.$$

Figure 9.6 illustrates that the vertices are on the major axis, a units from the center. The foci are on the major axis, c units from the center. For both equations, $b^2 = a^2 - c^2$. Equivalently, $c^2 = a^2 - b^2$.

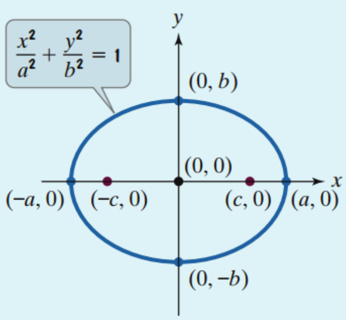


Figure 9.6(a) Major axis is horizontal with length $2a$.

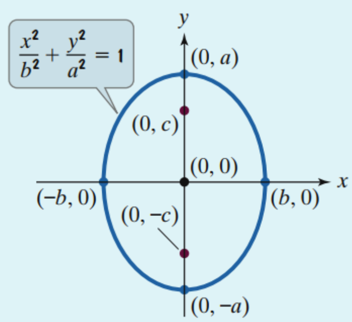


Figure 9.6(b) Major axis is vertical with length $2a$.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

***x*-intercepts: Set *y* = 0.**

$$\frac{x^2}{a^2} = 1$$
$$x^2 = a^2$$
$$x = \pm a$$

x-intercepts are $-a$ and a. The graph passes through $(-a, 0)$ and $(a, 0)$, which are the vertices.

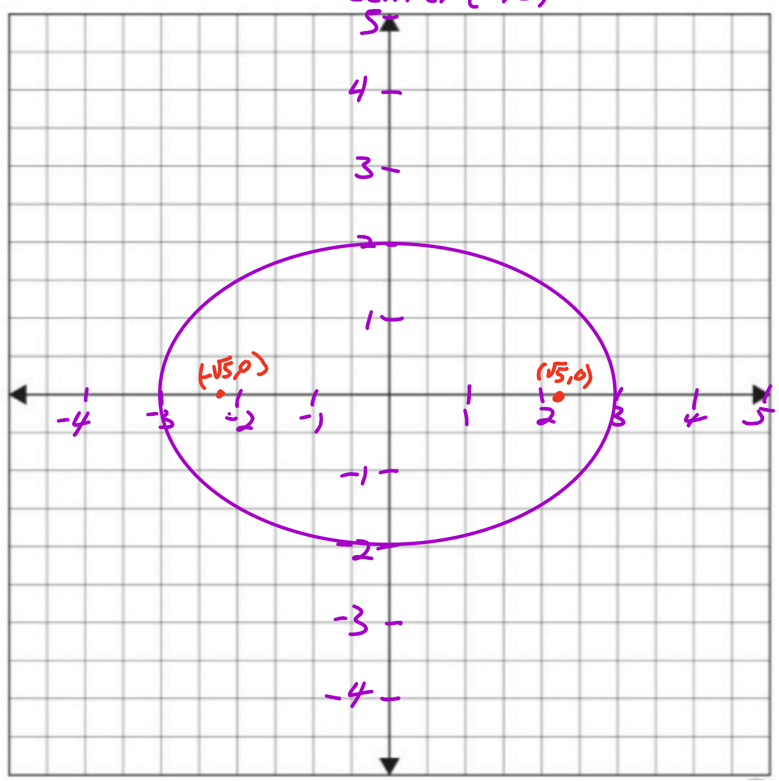
***y*-intercepts: Set *x* = 0.**

$$\frac{y^2}{b^2} = 1$$
$$y^2 = b^2$$
$$y = \pm b$$

y-intercepts are $-b$ and b. The graph passes through $(0, -b)$ and $(0, b)$.

Graph and locate the foci: $\frac{x^2}{9} + \frac{y^2}{4} = 1.$

center $(0,0)$



$\sqrt{9}$ = Semi major = 3

$\sqrt{4}$ = Semi minor = 2

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

(h,k) = center

Foci

$F^2 + 2^2 = 3^2$

$F^2 + 4 = 9$

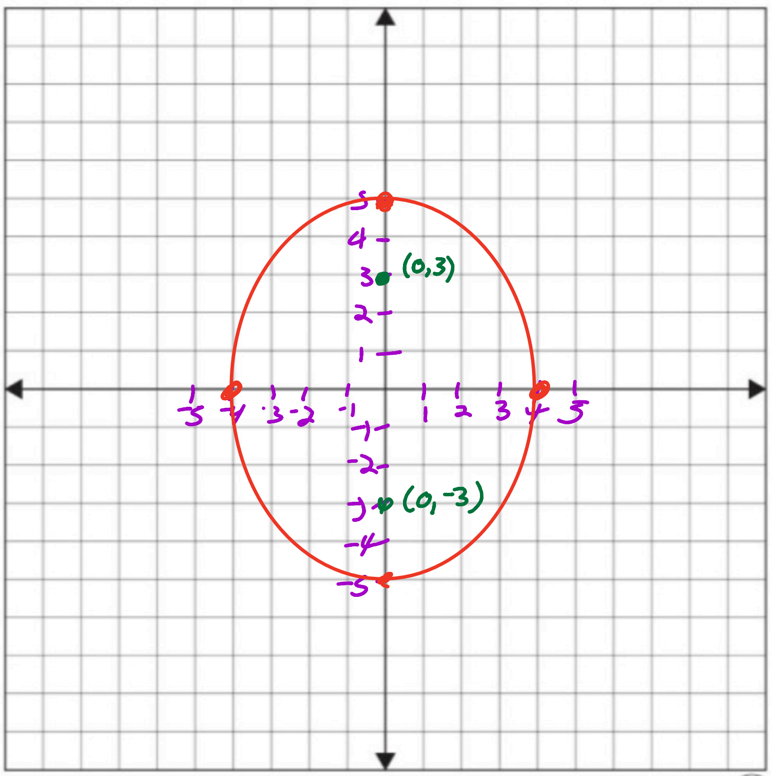
$F^2 = 5$

$F = \sqrt{5} = 2.23606$

Major axis (Bigger)

Minor axis (Smaller)

Graph and locate the foci: $25x^2 + 16y^2 = 400.$ $\frac{25x^2 + 16y^2}{400} = \frac{400}{400} \Rightarrow \frac{25x^2}{400} + \frac{16y^2}{400} = 1 \Rightarrow \frac{\cancel{25}x^2}{\cancel{25}\cdot 16} + \frac{\cancel{16}y^2}{25\cdot\cancel{16}} = 1$



$\frac{x^2}{16} + \frac{y^2}{25} = 1$

Center (0,0)

$\sqrt{16} = 4$

$\sqrt{25} = 5$

$F^2 + 4^2 = 5^2$

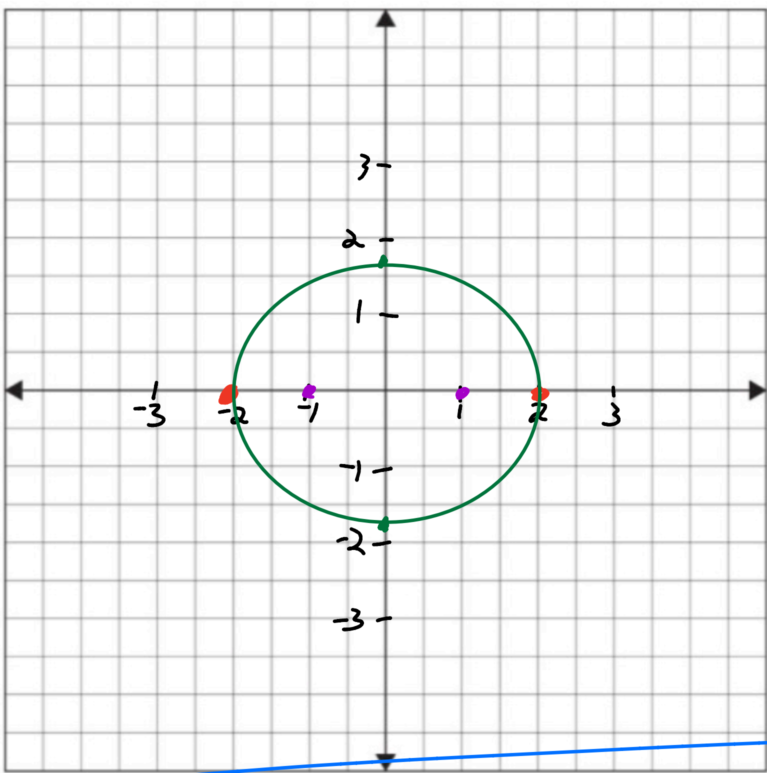
$F^2 + 16 = 25$

$F^2 = 9$

$F = 3$

Find the standard form of the equation of an ellipse with foci at $(-1, 0)$ and $(1, 0)$ and vertices $(-2, 0)$ and $(2, 0)$.

on major



$(\text{Foci Length})^2 + \text{semiminor}^2 = \text{semimajor}^2$

$1^2 + \text{semiminor}^2 = 2^2$

$1 + \text{semiminor}^2 = 4$

$\text{Semiminor}^2 = 3$

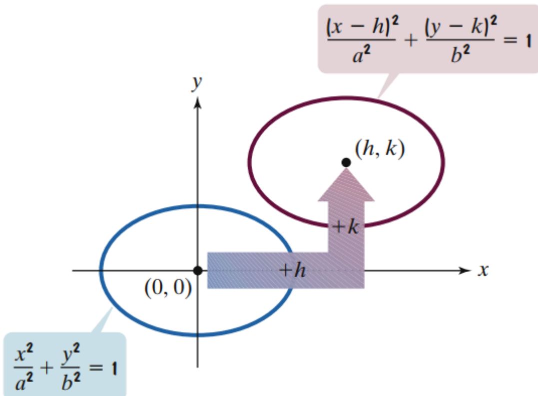
$\text{Semiminor} = \sqrt{3} = 1.73$

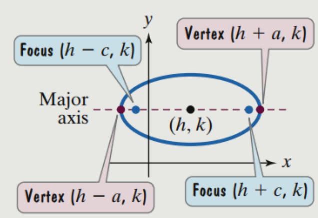
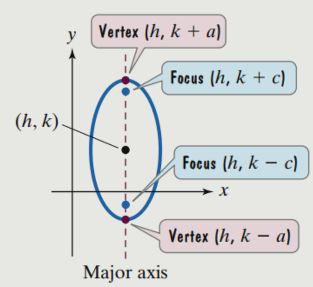
center $(0,0)$

$$\frac{(x-0)^2}{2^2} + \frac{(y-0)^2}{(\sqrt{3})^2} = 1$$

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad \text{and} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



<table>
<tr><th>Equation</th><th>Center</th><th>Major Axis</th><th>Vertices</th><th>Graph</th></tr>
<tr>
<td>$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
$a^2 > b^2$
Endpoints of major axis are a units right and a units left of center.
Foci are c units right and c units left of center, where $c^2 = a^2 - b^2$.</td>
<td>(h, k)</td>
<td>Parallel to the x-axis, horizontal</td>
<td>$(h - a, k)$
$(h + a, k)$</td>
<td>
</td>
</tr>
<tr>
<td>$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$
$a^2 > b^2$
Endpoints of the major axis are a units above and a units below the center.
Foci are c units above and c units below the center, where $c^2 = a^2 - b^2$.</td>
<td>(h, k)</td>
<td>Parallel to the y-axis, vertical</td>
<td>$(h, k - a)$
$(h, k + a)$</td>
<td>
</td>
</tr>
</table>

$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1.$$ Where are the foci located?

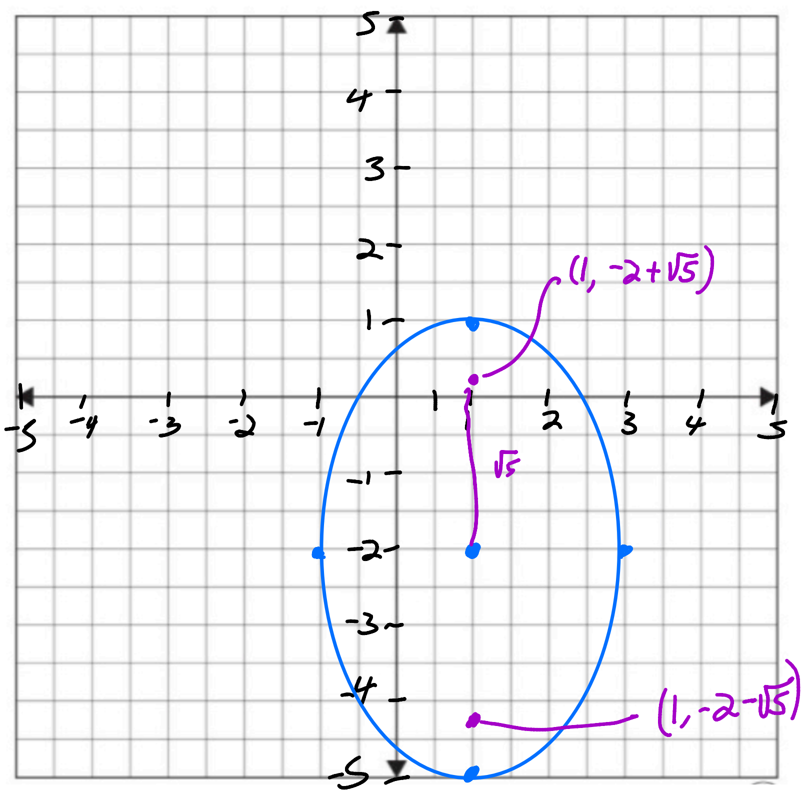
Left Right

$\sqrt{4} = 2$ = semiminor

$\sqrt{9} = 3$ = semi major

up/Down

Center=(1,-2)



Foci

$F^2 + 2^2 = 3^2$

$F^2 + 4 = 9$

$F^2 = 5$

$F = \sqrt{5} = 2.23$

Review completing the square

Complete the square for the binomial and factor the resulting perfect square trinomial.

$x^2 - 6x + 9$

$a = 1$

$b = -6$

$\frac{b}{2} = \frac{-6}{2} = -3$

$\left(\frac{b}{2}\right)^2 = (-3)^2 = 9$

$$9x^2 + 4y^2 - 18x + 16y - 11 = 0.$$

$+11 \quad +11$

$9x^2 - 18x + 4y^2 + 16y = 11$

$9(x^2 - 2x + 1) + 4(y^2 + 4y + 4) = 11 + 9 + 16 \Rightarrow 9(x-1)^2 + 4(y+2)^2 = 36$

$a = 1$, $b = -2$, $\frac{b}{2} = \frac{-2}{2} = -1$, $\left(\frac{b}{2}\right)^2 = (-1)^2 = 1$

$a = 1$, $b = 4$, $\frac{b}{2} = \frac{4}{2} = 2$, $\left(\frac{b}{2}\right)^2 = 2^2 = 4$

$$\frac{9(x-1)^2}{36} + \frac{4(y+2)^2}{36} = 1$$

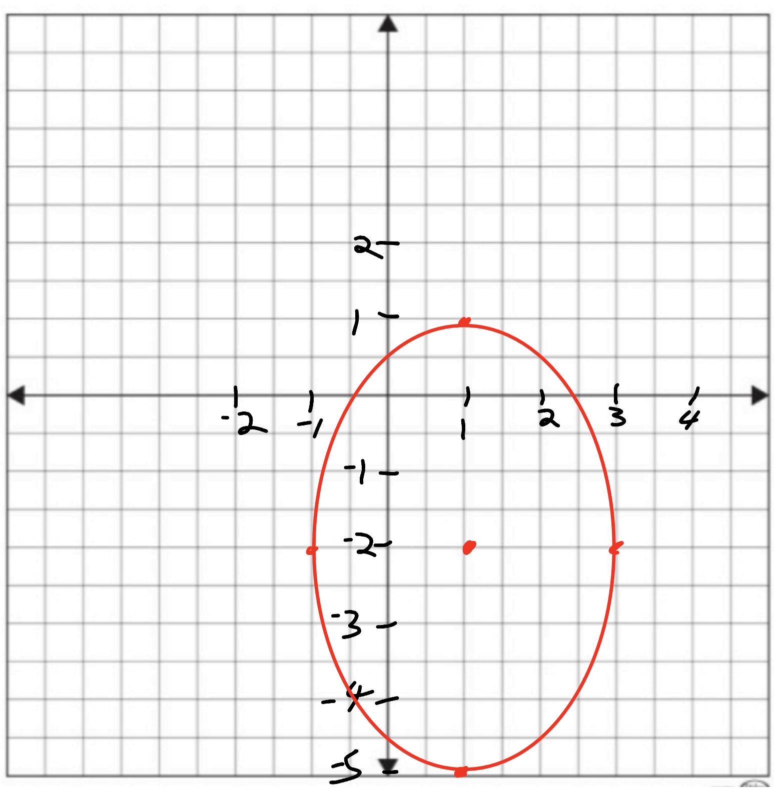
$$\frac{\cancel{9}(x-1)^2}{\cancel{9} \cdot 4} + \frac{\cancel{4}(y+2)^2}{9 \cdot \cancel{4}} = 1$$

$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$$

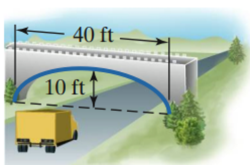
$(1,-2)$ center

semi major 3

semi minor $= 2$



A semielliptical archway over a one-way road has a height of 10 feet and a width of 40 feet (see **Figure 9.12**). Your truck has a width of 10 feet and a height of 9 feet. Will your truck clear the opening of the archway?



$$\frac{x^2}{20^2}+\frac{y^2}{10^2}=1$$

$$\frac{x^2}{400}+\frac{y^2}{100}=1$$

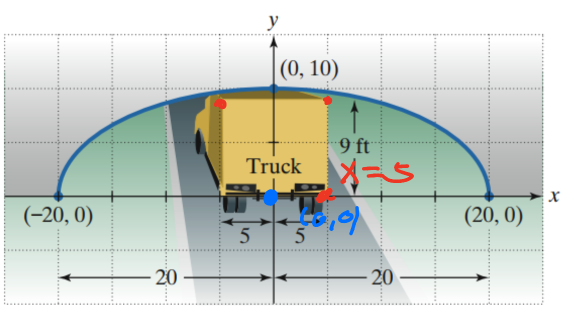


Figure 9.13

$$\frac{(5)^2}{400}+\frac{y^2}{100}=1$$

$$\frac{25}{400}\cancel{-\frac{25}{400}}+\frac{y^2}{100}=\frac{400}{400}-\frac{25}{400}$$

$$\cancel{100}\cdot\frac{y^2}{\cancel{100}}=\frac{375}{400}\cdot 100$$

$$\sqrt{y^2}=\sqrt{\frac{375}{4}}$$

$$y=\frac{\sqrt{375}}{2}=9.68 \text{ Feet}$$

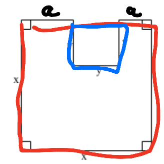
$$25(x-2)^2+81(y-5)^2=1 \Leftrightarrow \frac{(x-2)^2}{\frac{1}{25}}+\frac{(y-5)^2}{\frac{1}{81}}=1$$

$$\frac{\frac{(x-2)^2}{1}}{\frac{1}{25}}=\frac{(x-2)^2}{1}\cdot\frac{25}{1}=25(x-2)^2$$

$\sqrt{\frac{1}{25}}=\frac{1}{5}$ semi major

$\sqrt{\frac{1}{81}}=\frac{1}{9}=$ semi minor

The figure to the right shows a square floor plan with a smaller square area that will accommodate a combination fountain and pool. The floor with the fountain-pool area removed has an area of 60 square meters and a perimeter of 36 meters. Find the dimensions of the floor and the dimensions of the square that will accommodate the fountain and pool.



$x + x + x + a + a + 3y = 36$

area $= x^2 - y^2 = 60$

$2a + y = x$

$x^2 - y^2 = 60$

$x = -\frac{1}{2}y + 9$

$3x + \underbrace{2a + y}_{x} + 2y = 36 \Rightarrow 4x + 2y = 36 \Rightarrow y = -2x + 18$

$x^2 - (-2x + 18)^2 = 60$

$x^2 - (4x^2 - 72x + 324) = 60$

$x^2 - 4x^2 + 72x - 324 = 60$

-60

Each length of the large square floor is 8 meters, and each length of the small square is 2 meters.

(Simplify your answer. Type integers or decimals.)

$\frac{-3x^2 + 72x - 384}{-3} = \frac{0}{-3}$

$x^2 - 24x + 128 = 0$

$2 \quad 64$

$-8 - 16 = -24$

$x^2 - 8x - 16x + 128$

$x(x-8) - 16(x-8)$

$(x-8)(x-16) = 0$

$x = 8$ or $x = 16$

$x = 8$

$4x + 2y = 36$

$4(16) + 2y = 36 \Rightarrow y = -14$

$\cancel{x \neq 16}$

$4(8) + 2y = 36$

$32 + 2y = 36$

$2y = 4 \Rightarrow y = 2$

Solve the system by the method of your choice.

$2x^2 + xy = 12$

$-1(x^2 + xy = -4)$

$\Rightarrow$

$2x^2 + xy = 12$

$-x^2 - xy = 4$

$x^2 + 0 = 16$

$x = 4$

$x = -4$

$(4)^2 + 4(y) = -4$

$+16 + 4y = -4$

$4y = -20$

$y = -5$

The solution set is $\{(-4,5),(4,-5)\}$.

(Type an ordered pair, using integers or fractions. ...)

$(-4)^2 + -4y = -4$

$16 - 4y = -4 - 16$

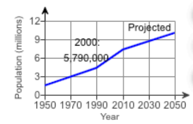
-16

$-4y = -20$

$y = 5$

a. In 2000, the population of a country was approximately 5.79 million and by 2050 it is projected to grow to 10 million. Use the exponential growth model $A = A_0 e^{kt}$, in which t is the number of years after 2000 and A_0 is in millions, to find an exponential growth function that models the data.

b. By which year will the population be 13 million?



$A = A_0 e^{kT}$

2000 Pop = 5.79 $T = 0$

$5.79 = A_0 e^{k \cdot 0} = A_0 e^{0} = A_0 \cdot 1$

$A_0 = 5.79$

2050 Pop = 10 $T = 50$

$A_0 = 5.79$

$$\frac{10}{5.79} = \frac{\cancel{5.79} \cdot e^{k \cdot 50}}{\cancel{5.79}}$$

$1.727115 = e^{50k}$

$\ln 1.727115 = \ln e^{50k} \Rightarrow \frac{.546452}{50} = \frac{\cancel{50} k \cdot \cancel{\ln e}^{\,1}}{\cancel{50}}$

$.010929056 = k$

$A = 5.79 e^{0.010929056 \cdot T}$

a. The exponential growth function that models the data is A = $\square$.

(Simplify your answer. Use integers or decimals for any numbers in th

$$\frac{13}{5.79} = \frac{\cancel{5.79} e^{0.010929056T}}{\cancel{5.79}}$$

$2.24525 = e^{0.010929056T}$

$\ln 2.24525 = \ln e^{0.010929056T} \Rightarrow \frac{0.808817}{0.010929056} = \frac{\cancel{0.010929056} T \cancel{\ln e}}{\cancel{0.010929056}}$

$74 = T$ years

Solve the system by the substitution method.

$$xy = 7 \Rightarrow y = \frac{7}{x}$$

$$x^2 + y^2 = 50$$

$$x^2 + \left(\frac{7}{x}\right)^2 = 50 \Rightarrow x^2 \cdot \left(x^2 + \frac{49}{x^2} = 50\right) \Rightarrow x^4 + 49 = 50x^2$$

$$-50x^2 \quad -50x^2$$

$$x^4 - 50x^2 + 49 = 0$$

$$1 \cdot 49 = 49$$

$$-1 \quad -49 = -50$$

$$x^4 - x^2 - 49x^2 + 49$$

$$x^2(x^2-1) - 49(x^2-1)$$

$$(x^2-1)(x^2-49) = (x-1)(x+1)$$

$$(x-7)$$

$$(x+7)$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The solution set is $\{(7,1),(-7,-1),(1,7),(-1,-7)\}$.
  (Type an ordered pair. Use a comma to separate answers as needed. Simplify your answer.)
- B. There is no solution.

$$y = \frac{7}{x} \qquad y = 7 = \frac{7}{1} \qquad y = \frac{7}{-1}$$

$$(1,7)(-1,-7)(7,1)(-7,-1)$$

$$x = \frac{7}{7} = 1 \qquad y = \frac{7}{-7} = -1$$

A planet follows an elliptical path described by $16x^2 + 4y^2 = 64$. A comet follows the parabolic path $y = x^2 - 4$. Where might the comet intersect the orbiting planet?

$$y + 4 = x^2$$

The comet might intersect the planet's orbit at the points $(\sqrt{3.2}, -0.8), (-\sqrt{3.2}, -0.8)$.
(Type ordered pairs. Use commas to separate answers.)

$$16x^2 + 4y^2 = 64$$

$$16(y+4) + 4y^2 = 64$$

$$16y + 64 + 4y^2 = 64$$

$$-64 \qquad -64$$

$$4y^2 + 16y = 0$$

$$4y(y+4) = 0$$

$$y = 0 \text{ or } y = -4$$

$$0 + 4 = x^2$$

$$4 = x^2$$

$$\pm 2 = x$$

$$(-2,0)(2,0)$$

$$-4 + 4 = x^2$$

$$0 = x^2$$

$$0 = x$$

$$(0,-4)$$

Solve the system by the substitution method.

$$x + y = 13 \Rightarrow x = 13 - y$$
$$(x+5)^2 + (y-9)^2 = 41$$

$\Rightarrow (13 - y + 5)^2 + (y-9)^2 = 41$

$(18-y)^2 + (y-9)^2 = 41$

$324 - 36y + y^2 + y^2 - 18y + 81 = 41$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

$2y^2 - 54y + 405 = 41$

$\quad\quad\quad\quad\quad -41 \quad -41$

A. The solution set is { (0,13),(− 1,14) }.
(Type an ordered pair. Use a comma to separate answers if needed.)

B. There is no solution.

$\dfrac{2y^2 - 54y + 364}{2} = \dfrac{0}{2}$

$y^2 - 27y + 182 = 0$

$-13 \cdot -14 = -27$

$x = 13 - y$

$y = 13$ or $y = 14$

$y^2 - 13y - 14y + 182 = 0$

$x = 13 - 13$ or $x = 13 - 14$

$x = 0 \quad x = -1$

$y(y-13) - 14(y-13) = 0$

$(y-13)(y-14) = 0$

$(0, 13) \quad (-1, 14)$

Solve the following system by the method of your choice.

$-9x + y = 36 \Rightarrow y = 36 + 9x$

$y = x^3 + 4x^2$

$\Rightarrow 36 + 9x = x^3 + 4x^2 - 9x - 36$

$-36 \; -9x$

The solution set is { (− 4,0), (− 3,9), (3,63) }.
(Type an ordered pair. Use a comma to separate answers as needed. Type exact answers, using radicals as needed. Simplify your answer.)

$y = 36 + 9(3)$

$36 + 27$

63

$0 = x^3 + 4x^2 - 9x - 36$

$x^2(x+4) - 9(x+4)$

$(x+4)(x^2 - 9)$

$0 = (x+4)(x-3)(x+3)$

$x = -4, 3, -3$

$y = 36 + 9x$

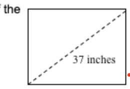
$36 + 9(-4)$

$36 - 36 = 0$

$36 + 9(-3)$

$36 - 27 = 9$

A rectangular painting has a diagonal measure of 37 inches and an area of 420 square inches. Use the formula for the area of a rectangle and the Pythagorean Theorem to find the length and width of the painting.



$x \cdot y = 420$

$x = \frac{420}{y}$

The length (the longer side) is 35 inches and the width (the shorter side) is 12 inches.
(Simplify your answers.)

$x^2 + y^2 = 37^2$

$x^2 + y^2 = 1369$

$y^2\left(\left(\frac{420}{y}\right)^2 + y^2\right) = 1369 \cdot y^2$

$176{,}400 + y^4 = 1369y^2$

$y = \cancel{-35}, \cancel{-12}, 12, 35$

Solve the system of equations by the substitution method.

$$y = x^2 - 7x - 5$$
$$y = -x^2 - 5x - 1$$

$\Rightarrow x^2 - 7x - 5 = -x^2 - 5x - 1$

$+x^2 + 5x + 1 \quad +x^2 + 5x + 1$

$\Rightarrow 2x^2 - 2x - 4 = 0$

$2 \cdot -4 = -8$

$-4 + 2 = -2$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- (●) **A.** The solution set is {(2, − 15),(− 1,3)}.
  (Type an ordered pair. Use a comma to separate answers if needed.)
- ( ) **B.** There is no solution.

$2x^2 - 4x + 2x - 4$

$2x(x-2) + 2(x-2)$

$(2x+2)(x-2) = 0$

$x = -1 \qquad x = 2$

$y = x^2 - 7x - 5$

$y = (-1)^2 - 7(-1) - 5$

$3 = 1 + 7 - 5$

$y = 2^2 - 7(2) - 5$

$4 - 14 - 5 = -15$

$3x^2 - 36x + \cancel{16} = 0$

$\quad\quad\quad\quad\;\; -\cancel{16} \quad -16$

$3x^2 - 36x = -16$

$a = 3$

Factor

$3(x^2 - 12x + 36) = -16 + 108 \Rightarrow 3(x-6)^2 = 92$

$a = 1$

$b = -12$

$\frac{b}{2} = -\frac{12}{2} = -6$

$\left(\frac{b}{2}\right)^2 = (-6)^2 = 36$